

## ROBUST CONTROLLER FOR AIRCRAFT YAW

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### ABSTRACT

In general, yaw control of aircraft plays a major role. It is controlled by rudder deflection by creating side force with help of rudder pedals. The aircraft yaw is controlled by several methods such as Fuzzy logic, linear quadratic controller (LQR), optimal pole placement method. These methodologies achieved only the transient response, but failed to obtain the steady state response. There are some cases, where the steady state response is necessary. In this research paper, we propose a new methodology called Linear Algebraic approach, where both the transient and steady state responses are achieved and the disturbance conditions are totally suppressed.

**KEYWORDS:** Aircraft, Flight Control, Two Parameter, Yaw, Rudder, Disturbance Suppression

### INTRODUCTION

After the first flights, articulated control surfaces were introduced for basic control which is operated by the pilot through a system of cables and pulleys. This technique survived for decades and at present used for small airplanes. The solution to this problem was made by the introduction of aero dynamic balances and tabs. But further growth of the aircraft and flight envelopes enhanced the need of powered systems to control the articulated aerodynamic surfaces.

Modern aircraft include a variety of automatic control system that aids the flight crew in navigation, flight management and augmenting the stability characteristic of the airplane. Designing an autopilot requires control system theory background and knowledge of stability derivatives at different altitudes and Mach numbers for a given airplane. Aircraft have a number of different control surfaces: the primary flight controls are pitch, roll and yaw controls which are controlled by deflection of elevators, ailerons and rudder. Yaw is controlled by the rudder. The pilot moves the rudder sideways and the necessary yaw angle to change the direction of the flight as per requirement of mission profile of the aircraft.

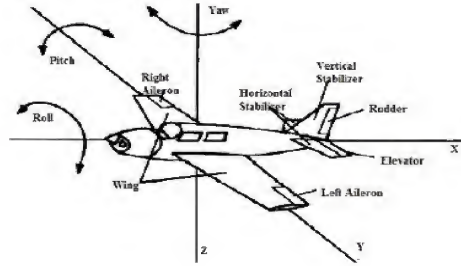
A modern controller (LQR) and intelligent fuzzy logic controller (FLC) is Developed for control the yaw of an aircraft system, but it failed to suppress the disturbances.

In this research paper, the control system design for yaw control is designed. A feed forward and feedback controller design is developed for control of the yaw of an aircraft system. In this the disturbances are totally suppressed based on the wind axis direction and flight path envelope.

### MODELING OF YAW CONTROL

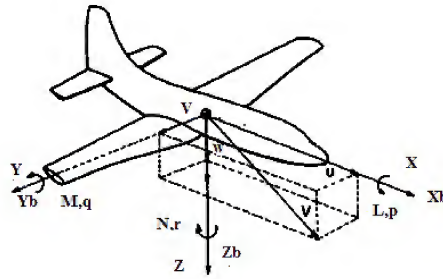
System Two types of dynamical equations are present for an aircraft. The lateral dynamic equations of motion, which represents the dynamics of aircraft with respect to lateral axis and longitudinal dynamic equations of motion which

represents the aircraft's dynamics with respect to longitudinal axis. Lateral dynamics includes yaw, roll and sideslip motions of aircraft. Pitching motion comes under longitudinal dynamics. In this research paper, control of yaw angle of aircraft, when it performs yawing motion is explained. The control surfaces of the aircraft are shown in Figure 1.



**Figure 1: Yaw, Roll & Pitch Motion of Aircraft [1]**

The forces, moments and Velocity components in the body Fixed frame of an aircraft system are as shown in figure 2, where  $L$ ,  $M$  and  $N$  represent the aerodynamic moment components, the term  $p$ ,  $q$  and  $r$  represent the angular rates components of roll, pitch and yaw axis and the term  $u$ ,  $v$  and  $w$  represent the velocity components of roll, pitch and yaw axis.



**Figure 2: Body Fixed Frame of Aircraft**

For deriving the lateral equations we assumed that the aircraft is in steady cruise with constant altitude and velocity. Also it is assumed that change in pitch angle does not change the speed of the aircraft and the reference flight conditions are symmetric with propulsive forces constant. Therefore,

$$v = p = q = r = \dot{\phi} = 0 \quad (1)$$

$$Y + mg C_{\theta} S_{\theta} = m \left( \frac{dv}{dt} + ru - pw \right) \quad (2)$$

$$L = I_x \frac{dp}{dt} - I_{xz} \frac{dr}{dt} + qr(I_z - I_y) - I_{xz} pq \quad (3)$$

$$N = -I_{xz} \frac{dp}{dt} + I_z \frac{dr}{dt} + pq(I_y - I_x) - I_{xz} qr \quad (4)$$

The equations are linearized using small- disturbance theory by replacing all the variables in equations (2), (3) and (4) with a reference value plus a small disturbance equation (5)

$$u = u_o + \Delta u; v = v_o + \Delta v; w = w_o + \Delta w$$

$$p = p_o + \Delta p; q = q_o + \Delta q; Y = Y_o + \Delta Y$$

$$r = r_o + \Delta r; L = L_o + \Delta L; M = M_o + \Delta M$$

$$\delta = \delta_o + \Delta\delta \quad (5)$$

$$\left(\frac{d}{dt} - Y_V\right)\Delta v - Y_P\Delta p + (u_o - Y_r)\Delta r - (g \cos \theta_o)\Delta\phi = Y_{\delta r}\Delta\delta_r \quad (6)$$

$$-L_V\Delta v + \left(\frac{d}{dt} - L_P\right)\Delta p - \left(\frac{l_{xz}}{I_x}\frac{d}{dt} + L_r\right)\Delta r = L_{\delta a}\Delta\delta_a + L_{\delta r}\Delta\delta_r \quad (7)$$

$$-N_V\Delta v + \left(\frac{d}{dt} - N_r\right)\Delta r - \left(\frac{l_{xz}}{I_z}\frac{d}{dt} + N_p\right)\Delta p = N_{\delta a}\Delta\delta_a + N_{\delta r}\Delta\delta_r \quad (8)$$

The lateral directional equations of motion consist of the side force, rolling moment and yawing moment equations of motion (6), (7) and (8) represents the linearized form. We are taking sideslip angle  $\Delta\beta$  instead of the side velocity  $\Delta v$ . These two quantities are related to each other in the following way, [3]

$$\Delta\beta \approx \tan^{-1}\left(\frac{\Delta\vartheta}{u_z}\right) \cong \frac{\Delta\vartheta}{u_z} \quad (9)$$

Lateral equations of motion in state space form is shown in equation (10)

$$\begin{bmatrix} \Delta\beta' \\ \Delta p' \\ \Delta r' \\ \Delta\phi' \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{u_o} & \frac{Y_P}{u_o} & -(1 - \frac{Y_r}{u_o}) & \frac{g \cos \theta_o}{u_o} \\ L_\beta & L_P & L_r & 0 \\ N_\beta & N_P & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix} + \begin{bmatrix} 0 & \frac{Y_{\delta a}}{u_o} \\ L_{\delta a} & L_{\delta r} \\ N_{\delta a} & N_{\delta r} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix} \quad (10)$$

For this system, the input will be the aileron deflection angle and the output will be the yaw angle. In this study, the data from NAVION Transport [2] is used in system analysis and modeling. The lateral directional derivatives stability parameters for this airplane are given in Table 1. The values are taken directly from reference [3], as these are standard data of the NAVION aircraft.

**Table 1: The Lateral Directional Derivatives Stability Parameters [3]**

General Aviation Airplane: NAVION	Y-Force Derivatives	Yawing Moment Derivatives	Rolling Moment Derivatives
Pitching Velocities	$Y_V = 0.254$	$N_V = 0.025$	$L_V = -0.091$
Side Slip Angle	$Y_\beta = -44.665$	$N_\beta = 4.549$	$L_\beta = -15.969$
Rolling Rate	$Y_P = 0$	$N_P = -0.349$	$L_P = -8.395$
Yawing Rate	$Y_r = 0$	$N_r = -0.76$	$L_r = 2.19$
Rudder Deflection	$Y_{\delta r} = 12.433$	$N_{\delta r} = -4.613$	$L_{\delta r} = 23.09$
Aileron Deflection	$Y_{\delta a} = 0$	$N_{\delta a} = -0.224$	$L_{\delta a} = -28.916$

The values in the above table is substituted in equation (10). The aileron deflection  $\delta_a$  in (10) is neglected as we are only concerned about rudder deflection  $\delta_r$ .

$$\begin{bmatrix} \Delta\beta' \\ \Delta p' \\ \Delta r' \\ \Delta\phi' \end{bmatrix} = \begin{bmatrix} -0.254 & 0 & -1 & 0.183 \\ -15.969 & -8.395 & 2.19 & 0 \\ 4.549 & -0.349 & -0.76 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix} + \begin{bmatrix} 0 \\ 23.09 \\ -4.613 \\ 0 \end{bmatrix} [\Delta\delta_r] \quad (11)$$

Transfer function from rudder deflection angle to yaw angle is given by the equation (12)

$$\frac{\Delta\phi(s)}{\Delta\delta_r(s)} = \frac{-4.6130s^3 - 47.9562s^2 - 11.8833s + 5.7410}{s^4 + 9.4090s^3 + 14.0189s^2 + 48.4991s + 0.3979} \quad (12)$$

### 3. Two-Parameter Controller Design

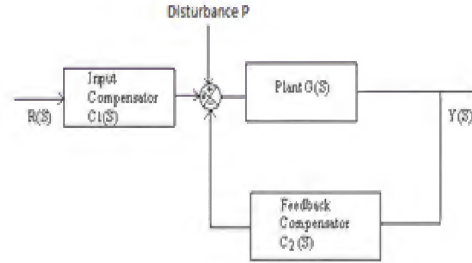


Figure 3: General Two Parameter Controller

The general two-parameter controller configuration is shown in Figure 3. The two-parameter controller configuration used in this work is shown in Figure 3. Here  $G(s)$  is plant transfer function,  $C_1(s)$  and  $C_2(s)$  are the two compensators, the two compensators will be chosen to have same denominator and have the form

$$C_1(s) = L(s)/A(s).$$

$$C_2(s) = M(s)/A(s). \quad (1)$$

Where  $L(s)$ ,  $M(s)$  and  $A(s)$  are polynomials to be determined. The transfer function from  $R$  to  $Y$  is

$$\begin{aligned} G_0(s) &= N(s) L(s) / [A(s) D(s) + M(s) N(s)] \\ &= N_0(s)/D_0(s) \end{aligned} \quad (2)$$

In equation (2) if all the roots of  $D_0(s)$  can be arbitrarily assigned, then the design is said to achieve arbitrary pole assignment. Assume  $G(s)$  as  $N(s)/D(s)$  which is strictly proper. Also  $N(s)$  and  $D(s)$  are co-prime. Then as per [5] arbitrary pole assignment is always possible. If the degree of the compensator  $C_2(s) < (n-1)$ , arbitrary pole placement is not possible for every set of poles. The design procedure for the implementation of the overall transfer function  $G_0(s)$  is as follows:

**Step 1:** Compute the following rational function

$$G_0(s) / N(s) = N_0(s) / \{ D_0(s) N(s) \} = \underline{N_p(s)} / \underline{D_p(s)} \quad (3)$$

Where  $N_p(s)$  and  $D_p(s)$  are co-prime. If  $N_o(s)$  and  $D_o(s)$  are co-prime then common factors may exist only between  $N_o(s)$  and  $N(s)$ . Cancel all common factors between them by computing  $G_0(s)/N(s)$ .

**Step 2:** From equation (3) the degree of  $A(s) D(s) + M(s) N(s)$  is  $\deg \{A(s)D(s)+M(s)N(s)\} = \deg\{A(s)\} + \deg\{D(s)\} = 2n-1$ . Since  $C_1(s)$  and  $C_2(s)$  are proper,  $\deg N(s) < \deg D(s)$  and  $\deg M(s) < \deg A(s)$ . If  $\deg D_p(s) = p < 2n-1$ , in order to get  $C_2(s)$  as proper and to match the coefficients of the same power of 's' on both sides of the equation, introduce an arbitrary polynomial  $D_p(s)$  of degree  $2n-1-p$  to get the total degree  $D_p(s) = (2n-1) = \deg \{A(s) D(s) + M(s) N(s)\}$ . Because this polynomial will be cancelled in the design procedure, its roots should be chosen in the acceptable pole zero cancellation regions. The cancelled roots i.e., the roots of  $D_p(s)$  must be chosen three to four times faster than the poles of  $G_0(s)$ . If the  $\deg D_p(s) = p = (2n-1)$ , set  $D_p(s) = 1$ . In most applications we have  $\deg D_p(s) < 2n-1$ , the case in which  $\deg D_p(s) > 2n-1$  is not considered. If  $G(s)$  is bipolar i.e.,  $\deg N(s)$  equals to  $\deg D(s)$ , then the above procedure in step 2 is modified as if  $\deg D_p(s) = p < 2n$ , introduce an arbitrary polynomial  $E(s)$  of degree  $(2n-p)$ .

**Step 3:**  $G_0(s) = \{N(s) N_p(s)\} / D_p(s)$

$$= N(s) \{E(s) N_p(s)\} / E(s) D_p(s) \quad (4)$$

Comparing (2) and (4) we get

$$L(s) = N_p(s) E(s) \quad (5)$$

$$A(s) D(s) + M(s) N(s) = E(s) D_p(s) = F(s) \quad (6)$$

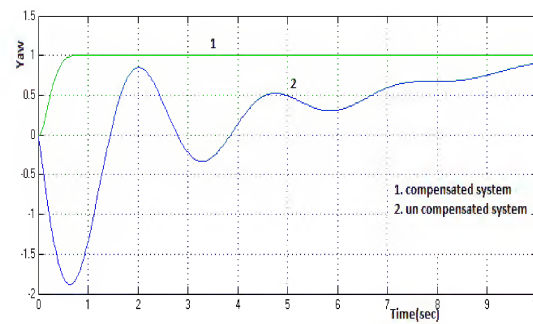
Equation (6) is called the Diophantine equation. In this equation  $D(s)$  and  $N(s)$  are known, the roots of  $D_p(s)$  are known as closed loop poles to be assigned from  $N(s)$  and the roots of  $E(s)$  are arbitrarily chosen. The polynomials  $A(s)$  and  $M(s)$  are found. We solve (6) by matching the coefficients of the same powers of 's' on both sides of the equation i.e., we solve a set of linear algebraic equations as in [4] to get  $A(s)$ ,  $M(s)$  and  $L(s)$ . Knowing these polynomials we can implement  $G_0(s)$ ,  $C_1(s)$  and  $C_2(s)$ . The introduction of  $E(s)$  in the design procedure is crucial. If we introduce  $E(s)$  as suggested in step 2, then  $M(s)/A(s)$  and  $L(s)/A(s)$  will be proper, and the resulting system is well posed. The design involves pole zero cancellations. The canceled poles are the roots of  $E(s)$ , which are chosen by the designer. Thus, if  $G_0(s)$  is stable and  $E(s)$  is Hurwitz, then the system is totally stable.

## SIMULATION AND RESULTS

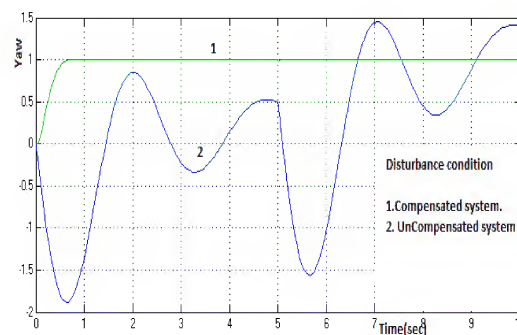
Simulations are first carried out on unity feedback system with and without disturbance. It is observed from Figure 4 although the steady state error is zero for a step input, and %overshoot is Zero for compensated system.

From Figure 5 we observe the compensated system totally suppressed disturbance at time 5 sec where as the Uncompensated system have large %over shoot and infinite steady state error





**Figure 4: Response for Without Disturbance Condition**



**Figure 5: Response for With Disturbance Condition**

## CONCLUSIONS

Thus, studies are carried out on aircraft yaw control, by incorporating two parameter controllers with linear algebraic method. The total system rejects the disturbance by tracking the reference input with zero steady state error. In general the procedure yields better results.

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